LEVEL II

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(18) SAMSO (19) TR-78-135

June 14, 1977

Refer to: 77-Y-30765

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Philadelphia, Pennsylvania 19101

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Subject: Contract (15) F04701-76-C-0181 (6) Comparison of Methodologies for Calculation of Loads Tranformations For Payloads With Redundant Interfaces.

Reference: 1) MCR-76-598, "Technical Operative Report (TOR), Stage II Depletion Shutdown Analysis, General Electric DSCS II/III," February 1977.

Attachments: A) Comparison of Loads Methodologies

B) Loads Transformation Development Using Unit Load Solution

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1. The enclosed attachments are submitted for your information and use. Attachment A details the analytical development and comparative results of a revised loads transformation development methodology. Attachment B details use of a unit load solution in conjunction with the new technology.

2. This information was previously provided informally to G.E. and Aerospace. MMC understands that G.E. is proceeding with the revised methodology to develop load transformations for the design loads cycle for DSCS II/III and DSCS III/III.

3. Any questions should be directed to Mr. Paul Jones, mail E-0971, telephone (303) 979-7000, extension 4250.

Very truly yours,

MARTIN MARIETTA CORPORATION

G. E. Reed
Project Engineer
Titan IIIC Launch Vehicles

SAMSO-TR-78-135

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ATTACHMENT A

COMPARISON OF LOADS METHODOLOGIES

I. SUMMARY

Payloads that fly on the Titan IIIC and on the T34D/IUS are constrained redundantly at up to eight bolt points on the interface ring. Due to this redundancy, inertial effects of the fairing and of the lower stages feed through the interface and induce additional loading in the payload. This letter documents comparisons between the present loads calculation method, which is dependent on upper body configuration, and the proposed new method, which is not. Two sample analyses were performed, one on a simple hypothetical problem and the other on the more complex DSCS III loads model from the GE DSCS II/III proposal study. It was the conclusion of this study that the new method may adequately replace the old.

II. PRESENT METHOD

Since the modal acceleration approach is highly desirable to minimize convergence problems, let us start with the discrete displacement loads transformation.

$$\left\{ ML \right\} = \left[T \right] \begin{Bmatrix} X_N \\ X_I \end{Bmatrix}$$

Where

X_N are non-interface payload displacements

X_I are interface payload displacements (maximum of 6 dof at 8 points = 48

Then, derive $\{x\}$ in terms of applied and inertia loading

$$\begin{Bmatrix} X_N \\ X_I \end{Bmatrix} = \left[\begin{array}{c|c} K_{NN} & K_{NI} \\ \hline K_{IN} & K_{II} + \overline{K}_{TS} \end{array}\right]^{-1} \begin{Bmatrix} P_N \\ P_I \end{Bmatrix} \tag{II.2}$$

Where

$\overline{K}_{TS}$ is the transtage stiffness reduced to its interface coordinates

However, the loading may be expressed in terms of mixed coordinate vector of cantilevered modal acceleration and discrete interface acceleration -

$$\begin{Bmatrix} P_N \\ P_I \end{Bmatrix} = - \begin{bmatrix} M_N & \\ & M_I \end{bmatrix} \begin{bmatrix} \phi_N & \phi_c \\ & I \end{bmatrix} \begin{Bmatrix} \ddot{\bar{q}}_P \\ \ddot{X}_I \end{Bmatrix} \tag{II.3}$$

Where

ϕ_N are normal cantilevered modes of the payload

ϕ_c are constraint modes of the payload

Expand equation II.2

$$\left\{ML\right\} = -\left[T\right]\left[\begin{array}{c|c} K_{NN} & K_{NI} \\ \hline K_{IN} & K_{II}+\bar{K}_{TS} \end{array}\right]^{-1}_{,}\left[\begin{array}{cc} M_N & \\ & M_I \end{array}\right]\left[\begin{array}{cc} \phi_N & \phi_c \\ & I \end{array}\right]\left\{\begin{array}{c} \ddot{\bar{q}}_{\delta P} \\ \ddot{X}_I \end{array}\right\} \tag{II.4}$$

and express in shorthand as

$$\left\{ML\right\} = \left[\,LTMQ \mid LTMX\,\right]\left\{\begin{array}{c} \ddot{\bar{q}}_{\delta P} \\ \ddot{X}_I \end{array}\right\} \tag{II.5}$$

Equation II.5 can be further expanded using the relationships resulting from modal coupling,

$$\left\{\begin{array}{c} \ddot{\bar{q}}_{\delta} \\ \ddot{X} \end{array}\right\} = \left[\begin{array}{c} \Phi_P \\ \phi_I \end{array}\right]\left\{\ddot{\xi}\right\} \quad ,$$

to yield the present modal loads transformation:

$$\left\{ML\right\} = \left[\,\underbrace{LTMQ \mid LTMX}_{\substack{\text{supplied by payload} \\ \text{contractor \underline{dependent} on} \\ \text{interface stiffness}}}\,\right]\left[\begin{array}{c} \Phi_P \\ \phi_I \end{array}\right]\left\{\ddot{\xi}\right\} \tag{II.6}$$

III. PROPOSED METHOD

In order to eliminate the dependency of LTM Development on interface stiffness (transtage, IUS, fairing, etc.), redefine the displacement vector of equation II.2 as follows:

$$\begin{Bmatrix} X_N \\ X_I \end{Bmatrix} = \underbrace{\left[\begin{array}{c|c} K_{NN}^{-1} & \\ \hline & \end{array}\right] \begin{Bmatrix} P_N \\ P_I \end{Bmatrix}}_{\substack{\text{Deflection relative to fixed} \\ \text{interface, i.e. due to applied} \\ \text{and inertial loading on payload}}} + \underbrace{\begin{bmatrix} \phi_c \\ I \end{bmatrix} \begin{Bmatrix} X_I \end{Bmatrix}}_{\substack{\text{Deflection due to interface} \\ \text{motion, i.e. applied and} \\ \text{inertial loading due to} \\ \text{fairing, transtage, P/L} \\ \text{interface, etc.}}} \qquad \text{(III.1)}$$

The loads equation (II.1) can be rewritten with equations III.1 and II.3 as -

$$\begin{Bmatrix} ML \end{Bmatrix} = - \begin{bmatrix} T \end{bmatrix} \left[\begin{array}{c|c} K_{NN}^{-1} & \\ \hline & \end{array}\right] \begin{bmatrix} M_N & \\ & M_I \end{bmatrix} \left[\begin{array}{c|c} \phi_N & \phi_c \\ \hline & I \end{array}\right] \begin{Bmatrix} \ddot{\bar{q}}_P \\ \ddot{X}_I \end{Bmatrix}$$

$$+ \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} \phi_c \\ I \end{bmatrix} \begin{Bmatrix} X_I \end{Bmatrix} \qquad \text{(III.2)}$$

Equation III.2 can be expressed in shorthand as -

$$\left\{ ML \right\} = \Big[\underbrace{LTMAQ \;\vert\; LTMAX}_{LT1} \Big] \left\{ \begin{matrix} \ddot{\tilde{q}}_P \\ \ddot{X}_I \end{matrix} \right\} + \Big[\underbrace{LTMDX}_{LT2} \Big] \left\{ X_I \right\} \tag{III.3}$$

where

$$\left[LTMAQ \right] = - \left[T \right] \left[K_{NN}^{-1} \; M_N \; \phi_N \right] \tag{III.4}$$

$$\left[LTMAX \right] = - \left[T \right] \left[K_{NN}^{-1} \; M_N \; \phi_C \right] \tag{III.5}$$

$$\left[LTMDX \right] = + \left[T \right] \begin{bmatrix} \phi_C \\ I \end{bmatrix} \tag{III.6}$$

From the modal coupling analysis, the acceleration vector can be transformed to coupled modal acceleration.

$$\left\{ \begin{matrix} \ddot{\tilde{q}}_P \\ \ddot{X}_I \end{matrix} \right\} = \begin{bmatrix} \Phi_P \\ \phi_I \end{bmatrix} \left\{ \ddot{\xi} \right\} \tag{III.7}$$

From both the transtage uncoupled vibration analysis and the modal coupling analysis, the interface displacement vector can be transformed to the coupled modal acceleration domain. Note that the vector X_I has both a rigid and an elastic component ($X_I = X_{I_R} + X_{I_E}$) but the effect on the loads due to the rigid component is zero for a free-free system.

$$\{X_I\} = [SEL]\,[E^*]\left(\cancelto{0}{\{F\}} + [ILT_{TS}]\{\ddot{\xi}\}\right) \qquad \text{(III.8)}$$

Where

SEL is a selector matrix to pick up interface dof (*note the coordinate system)

E^* is the constrained transtage flexibility matrix with applicable stiffness loading (PL, fairing, etc.)

ILT_{TS} is an inertial loads transformation for the <u>mass loaded</u> transtage (form of $-[M][\phi]$).

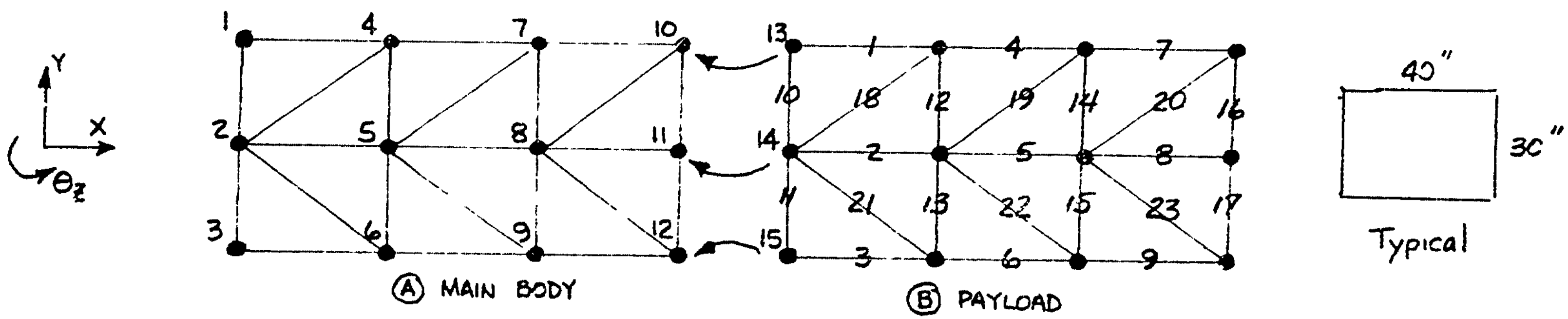
Using these relationships, the proposed modal loads transformation takes the form of:

$$\left\{ML\right\} = \left(\underbrace{\left[LTMAQ \middle| LTMAX\right]}_{\substack{\text{from payload} \\ \text{contractor \underline{independent}} \\ \text{of interface stiffness}}} \underbrace{\begin{bmatrix} \Phi_P \\ \phi_I \end{bmatrix}}_{\substack{\text{developed in} \\ \text{modal coupling} \\ \text{analysis}}} + \underbrace{\left[LTMDX\right]}_{\substack{\text{from payload} \\ \text{contractor \underline{independent}} \\ \text{of interface stiffness}}} \underbrace{\left[SEL\right]\left[E^*\right]}_{\substack{\text{from transtage} \\ \text{uncoupled vibration} \\ \text{analysis}}} \underbrace{\left[ILT_{TS}\right]}_{\substack{\text{developed in modal} \\ \text{coupling analysis/} \\ \text{includes effects of} \\ \text{payload (\&fairing if} \\ \text{applicable) as well as} \\ \text{transtage inertia}}} \right) \left\{\ddot{\xi}\right\} \tag{III.9}$$

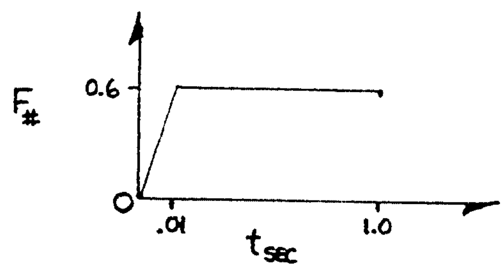
IV. EXAMPLE PROBLEMS

IV.1 SIMPLE TRUSS

For this problem, the following example structure was used:



Body (B) represents a payload while Body (A) represents transtage, IUS, etc. (B) ties to (A) at points 10/13, 11/14, 12/15. Both bodies are pinned joint structures, 2 dof at each joint with 3 rigid body modes for the composite structure. All members are axial bars (k = 40#/in). Mass at joints 7, 8, 10, 11 is 0.10 #sec^2/in, the rest 0.01 #sec/in. The ramp forcing function below is applied in x-direction, joint 2, Truss A.



1. <u>Exact Solution</u> - The mass and stiffness matrices of (A) and (B) were tied together discretely to give the free-free equations of motion. All modes of the A/B system were kept in generating the coefficients. Member load time histories were then solved for using the form:

$$\{L(t)\} = [K_B][\psi_B][E_{AB}]\left(\{F(t)\} - [M_{AB}][\phi_{AB}]\{\ddot{\xi}(t)\}\right)$$

and tabulated in Table 1 as a basis of comparison.

2. <u>Old Method</u> - Bodies (A) and (B) were inertially modally coupled using all component modes; the coupled system was excited by the ramp transient, yielding time histories of modal and discrete acceleration. Loads were calculated according to Equation II.6.

$$\{L(t)\} = \left[LTMQ \,\vdots\, LTMX\right]\begin{bmatrix}\Phi_P \\ \phi_I\end{bmatrix}\{\ddot{\xi}(t)\}$$

Table 1 tabulates results using 30 and 42 (all) modes.

3. New Method - Everything same as old method except that loads were calculated according to Equation III.9:

$$\{L(t)\} = \Bigg(\underbrace{\left[LTMAQ \,\vdots\, LTMAX \right]}_{LT1} \begin{bmatrix} \Phi_P \\ \phi_I \end{bmatrix} + \underbrace{\left[LTMDX \right]}_{LT2} \left[SEL \right] \left[E^* \right] \left[ILT_{TS} \right] \Bigg) \{\ddot{\xi}(t)\}$$

Table 1 tabulates results using 30 and 42 (all) modes.

IV.2 DSCS III

For this much more complex case, use was made of the modes, frequency and transient response data of the GE DSCS II/III loads report of Reference 1. Basically, the coupled configuration consisted of a DSCS II/DSCS III satellite cluster, a transtage and a burned out Stage II booster. The system was excited by a fuel-leading depletion, B-8, from the set of measured flight data transients commonly used in our analyses. Three distinct types of loads are tabulated in Table 2:

1. One-g - The loads resulting from the uniform application of the rigid body modal equivalent of a unit g-level axial acceleration.

2. <u>Old Method</u> - The loads reported in Reference 1, consistent with Equation II.6.

3. <u>New Method</u> - The loads according to Equation III.9.

Mention should be made here of two abbreviations used in Table 2. The expression "W Tanks" indicates standard Equation III.9. The expression "W/O Tanks" points to an analytical device in which the large inertial effects or the transtage tanks on the payload loads are deleted, i.e. the loads are calculated by a modified Equation III.9:

$$\left\{L(t)\right\} = \left(\left[LTMAQ \mid LTMAX \right] \begin{bmatrix} \Phi_P \\ \phi_I \end{bmatrix} + \left[LTMDX \right] \left[SEL \right] \left[E^* \right] \begin{bmatrix} ILT_{\text{NO TANKS}} \\ \hline 0 \end{bmatrix} \right) \left\{ \ddot{\xi}(t) \right\}$$

V. CONCLUSIONS

From the simple truss results, it is apparent that the old and new methods produce different answers. It should be noted that these differences *are not* attributable to the use of modal coordinates since the differences appear in the "all modes" case. It was expected that this example problem would in fact emphasize differences between the two methods since the model was set up to contain large mass assymetry in Body A. The results indicate that the feedback from such assymetric mass is a shortcoming of the old method.

From the DSCS III loads results, it is also apparent that the methods produce different answers. At load stations away from the interface, the disagreement is slight. At or near the interface, the disagreement increases due to feedback differential. The transtage model used here has large assymetric masses in the form of propellant tanks which have rigid and elastic modes with the tanks moving both in and out of phase with each other. The elimination of the inertial feedback of these tanks in the "W/O TKS" case produced results much closer to those of the old method, further confirming the above conclusion.

In general, the following observations may be made:

1. The LT1/LT2 approach (New Method) produces technically correct answers.

2. LT1 and LT2 may be formed by the payload contractor without necessarily knowing what type of structure the payload will interface with.

3. Due to the final form of LT1 and LT2, it does not matter whether or not there are more structures beyond the interface structure (i.e. other booster stages, fairing).

4. The form of LT1 is quite similar to the old method transformation; only the "E" used in its formation is different (and more convenient for the payload contractor to form).

5. The inclusion of LT2 into the loads computations assures us that additional loads, due to applied or inertial loads on the lower structure, will be included; i.e. LT2 is a feedback term which potentially produces additional loads which the old method does not have the capability to include.

TABLE 1

LARGEST VALUE - MEMBER LOADS (LBS)

MEMBER NO.	EXACT SOLUTION	ALL (42) MODES LT1/LT2	ALL (42) MODES OLDTRAN	30 MODES LT1/LT2	30 MODES OLDTRAN
* 1	.0956	.0956	.1105	.0950	.1100
* 2	-.0597	-.0597	-.0747	-.0581	-.0725
* 3	-.1919	-.1919	-.1757	-.1980	-.1900
4	.0811	.0811	.0852	.0812	.0854
5	-.0690	-.0690	-.0722	-.0664	-.0698
6	-.1232	-.1232	-.1160	-.1238	-.1170
7	-.0581	-.0581	-.0539	-.0561	-.0520
8	-.0667	-.0667	-.0667	-.0647	-.0647
9	-.1229	-.1229	-.1213	-.1265	-.1248
* 10	.0825	.0825	.0278	.0824	.0289
* 11	-.0666	-.0666	-.0227	-.0674	-.0235
12	-.0616	-.0616	-.0464	-.0632	-.0425
13	-.0766	-.0766	-.0569	-.0722	-.0522
14	.0457	.0457	.0430	.0477	.0445
15	-.0506	-.0506	-.0445	-.0491	-.0457
16	.0337	.0337	.0322	.0359	.0344
17	-.0314	-.0314	-.0293	-.0298	-.0280
* 18	-.0656	-.0656	-.0577	-.0677	-.0577
19	-.0562	-.0562	-.0544	-.0535	-.0499
20	-.0602	-.0602	-.0585	-.0557	-.0540
* 21	.0959	.0959	.0764	.0906	.0763
22	.0650	.0650	.0542	.0641	.0532
23	.0609	.0609	.0564	.0614	.0589

* = MEMBERS TOUCHING INTERFACE JOINTS

TABLE 2
COMPARE DSCS III LOADS METHODS
FORCING FUNCTION FB-8

	OLD 1G(X) VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT	OLD MAX VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT	OLD MIN VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT
D3L1	-336.24	100	100	9.35	101	101	-1375.86	100	100
D3L2	118.08	100	100	454.09	100	100	-9.90	100	101
D3L3	137.60	99	100	768.31	100	100	-73.64	103	101
D3L4	-1.32	118	96	3.47	88	103	-10.04	104	99
D3L5	-33.01	96	101	55.72	103	99	-188.19	100	101
D3L6	-80.21	100	100	6.87	101	102	-303.85	100	100
D3L7	-350.24	100	100	11.20	99	99	-1357.88	100	100
D3L8	-119.32	101	100	29.21	98	101	-513.18	100	100
D3L9	137.78	100	100	577.82	100	100	-20.25	91	85
D3L10	1.15	121	97	9.95	100	100	-4.47	92	104
D3L11	-26.38	96	101	62.12	104	100	-185.36	99	100
D3L12	79.53	100	100	313.60	100	100	-39.45	100	100
D3L13	-335.11	100	100	31.45	102	102	-1273.54	100	100
D3L14	-112.61	101	100	59.72	101	102	-521.17	100	100
D3L15	-136.54	99	100	141.62	102	101	-656.85	100	100
D3L16	-1.19	113	96	3.76	95	104	-5.88	105	99
D3L17	25.56	96	100	138.63	99	100	-43.95	105	101
D3L18	81.50	98	100	340.26	99	100	-17.93	102	100
D3L19	-347.18	100	100	27.32	97	97	-1313.69	100	100
D3L20	113.84	101	100	452.60	100	100	-9.02	103	103
D3L21	-138.84	100	100	33.87	99	97	-540.41	100	100
D3L22	1.50	113	97	7.88	105	100	-5.35	95	102
D3L23	33.01	98	101	168.53	99	100	-49.13	102	98
D3L24	-80.29	98	101	10.49	102	101	-310.86	99	100
D3L25	337.12	100	100	1381.02	100	100	-9.86	101	101
D3L26	10.50	81	104	64.47	96	103	-22.05	122	101
D3L27	-179.82	95	101	2.19	129	82	-722.33	97	101
D3L28	—			—			—		
D3L29	-15.48	93	101	3.25	141	102	-74.69	97	101
D3L30	6.41	93	101	30.91	97	101	-1.35	141	102
D3L31	—			—			—		
D3L32	16.89	93	102	70.73	96	101	-1.87	178	79
D3L33	6.99	93	102	29.28	96	101	-.77	178	79
D3L34	348.53	100	100	1359.08	100	100	-9.95	97	97
D3L35	-14.57	87	103	3.77	144	94	-61.75	93	102

	OLD 1G (X) VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT	OLD MAX VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT	OLD MIN VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT
D3L36	-225.47	97	101	5.83	107	93	-869.99	98	100
D3L37	-66.34	101	100	11.30	97	101	-290.21	100	100
D3L38	78.54	111	98	378.72	105	99	-30.26	62	103
D3L39	-67.55	100	100	10.82	100	101	-293.07	100	100
D3L40	68.06	101	100	277.56	100	100	-2.52	94	96
D3L41	11.20	97	101	46.52	99	100	-2.64	121	99
D3L42	148.25	96	101	633.81	98	101	-55.57	120	98
D3L43	63.56	101	100	272.92	100	100	-21.26	100	102
D3L44	10.53	97	101	50.72	98	100	-6.23	105	100
D3L45	-137.64	95	101	95.58	105	100	-690.37	98	101
D3L46	-62.11	101	100	29.59	100	102	-280.52	101	100
D3L47	73.99	112	97	349.00	106	99	-53.06	83	111
D3L48	63.91	100	100	284.99	100	100	-28.00	101	102
D3L49	333.66	100	100	1249.08	100	100	-30.59	103	103
D3L50	-6.16	92	99	14.01	111	105	-31.10	96	98
D3L51	213.74	96	101	787.92	98	100	-17.92	107	103
D3L52	—			—			—		
D3L53	8.36	102	97	53.45	101	99	-25.00	101	104
D3L54	-3.46	102	97	10.35	101	104	-22.13	101	99
D3L55	—			—			—		
D3L56	-9.50	101	100	5.84	101	103	-40.22	102	101
D3L57	-3.93	101	100	2.42	101	103	-16.65	102	101
D3L58	344.62	100	100	1310.46	100	100	-24.90	97	97
D3L59	5.39	86	104	32.38	96	101	-9.41	117	97
D3L60	183.03	95	101	678.82	97	101	-15.65	104	98
D3L61	-63.05	101	100	3.55	103	104	-251.39	101	100
D3L62	-75.15	112	97	29.38	41	114	-293.95	107	98
D3L63	64.72	100	100	258.38	100	100	-3.16	103	103
D3L64	65.61	101	100	250.05	100	100	-3.62	98	98
D3L65	-8.70	95	101	2.09	121	96	-33.16	97	101
D3L66	-104.99	92	102	53.49	116	97	-436.34	96	101
D3L67	64.75	100	100	258.97	100	100	-5.04	100	101
D3L68	-8.60	95	101	2.94	131	96	-40.27	98	100
D3L69	102.54	92	101	548.44	96	101	-95.27	119	98
D3L70	-65.63	100	100	5.55	100	101	-255.05	100	100
D3L71	-77.24	110	98	34.56	57	116	-336.18	106	100
D3L72	-66.72	100	100	5.69	100	100	-260.30	100	100
D3L73	-939.48	100	100	-9.51	109	113	-3511.81	100	100

	OLD 1G(X) VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT	OLD MAX VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT	OLD MIN VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT	
D3L74	15.43	-395	185	197.29	41	120	-212.67	170	91	
D3L75	-2.27	2926	746	225.93	160	89	-148.38	47	131	
D3L76	-883.00	101	100	54.50	96	96	-3085.18	100	100	
D3L77	17.40	-389	222	312.82	52	109	-164.25	187	77	
D3L78	1.35	-5198	1154	269.86	54	108	-158.72	170	83	
D3L79	-842.16	100	100	74.03	104	103	-2954.91	100	100	
D3L80	-16.62	-428	226	153.04	216	80	-290.37	40	116	
D3L81	2.69	-2760	491	238.92	33	107	-149.02	198	85	
D3L82	-878.41	101	100	8.44	71	73	-3336.62	100	100	
D3L83	-16.22	-354	183	215.12	163	86	-211.59	32	109	
D3L84	-1.78	-4403	671	210.35	176	94	-161.77	13	114	
D3L85	431.67	115	97	1628.72	109	97	-11.58	28	101	
D3L86	265.22	115	97	999.70	109	98	-7.83	39	104	
D3L87	-225.53	115	97	6.27	35	105	-852.40	108	97	FOOT
D3L88	-296.92	110	92	4.26	-20	122	-1139.13	101	91	8F
D3L89	338.51	105	91	1307.25	97	89	-10.47	83	139	
D3L90	-316.95	118	98	-1.14	1139	563	-1220.07	108	98	
D3L91	507.81	86	102	1883.09	92	102	-11.91	163	115	
D3L92	-280.65	87	103	6.06	168	117	-1042.42	92	102	
D3L93	227.79	84	103	841.92	91	103	-6.90	159	114	FOOT
D3L94	308.14	102	92	1175.91	97	91	6.76	122	79	4F
D3L95	424.32	107	90	1629.90	99	89	3.69	424	242	
D3L96	-312.11	105	98	-6.25	121	88	-1196.28	100	96	
D3L97	478.11	85	104	1750.28	92	102	-12.13	146	95	
D3L98	-265.38	86	104	7.48	141	95	-970.03	92	103	
D3L99	-213.76	85	104	3.64	172	92	-790.08	92	103	FOOT
D3L100	-304.89	80	97	28.08	110	96	-1052.05	86	96	32F
D3L101	-413.91	91	89	48.61	100	94	-1430.68	95	99	
D3L102	-317.02	92	104	27.58	109	98	-1107.49	94	101	
D3L103	404.88	118	96	1454.21	108	96	-44.79	81	97	
D3L104	247.97	119	96	893.62	108	96	-27.47	80	97	
D3L105	212.40	119	98	764.66	109	97	-24.25	82	97	FOOT
D3L106	293.33	115	109	1045.80	110	107	-38.54	95	97	29F
D3L107	-337.36	117	118	51.76	96	94	-1234.28	112	112	
D3L108	-315.21	111	103	37.84	88	93	-1090.97	107	102	
D3L109	386.14	118	95	1327.46	111	95	-37.32	70	109	FOOT
D3L110	-237.52	117	94	22.69	73	113	-816.24	110	94	24F

	OLD 1G (X) VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT	OLD MAX VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT	OLD MIN VALUE	W/TANKS NEW/OLD PCT	W/O TKS NEW/OLD PCT	
D3L111	202.26	119	97	694.68	112	97	-20.86	71	105	
D3L112	-264.88	135	124	40.39	66	71	-907.11	128	121	FOOT
D3L113	-308.22	131	132	56.90	63	64	-1048.38	126	127	24F
D3L114	284.79	127	114	980.38	122	114	-37.91	100	107	
D3L115	456.02	84	104	1627.45	91	103	-48.86	118	104	
D3L116	254.13	81	103	906.02	88	102	-27.69	118	102	
D3L117	-204.94	81	103	21.73	118	102	-733.67	88	102	FOOT
D3L118	289.95	76	98	1010.36	88	98	-36.15	126	111	21F
D3L119	-378.74	114	118	49.06	116	115	-1308.75	115	117	
D3L120	318.18	49	81	1111.92	59	81	-43.12	332	99	
D3L121	474.83	86	103	1790.45	92	102	2.40	-395	132	
D3L122	264.56	87	101	999.14	92	101	2.58	-102	139	
D3L123	213.37	81	101	801.52	88	101	-2.78	304	41	FOOT
D3L124	-300.34	125	102	-3.61	297	57	-1166.65	118	103	15F
D3L125	395.56	145	113	1557.42	132	113	-5.39	-332	-63	
D3L126	329.02	107	85	1280.51	103	90	4.83	-7	-183	
D3L127	403.58	117	97	1572.53	110	98	-22.25	68	104	
D3L128	-248.34	116	96	14.47	74	107	-968.35	109	97	
D3L129	-211.59	118	97	11.89	68	106	-826.98	110	97	FOOT
D3L130	277.75	135	101	1100.33	122	102	-12.20	15	93	13F
D3L131	325.69	136	97	1313.37	121	98	-20.30	46	109	
D3L132	296.80	137	108	1174.90	126	109	-6.67	-93	65	

ATTACHMENT B

LOADS TRANSFORMATION DEVELOPMENT

USING UNIT LOAD SOLUTION

UNIT LOAD SOLUTION

In developing loads transformations with the new method of Attachment A, a unit load solution is offered below which avoids large order matrix manipulation. Start with the discrete displacement loads transformation:

$$\left\{ ML \right\} = \left[T \right] \left\{ \begin{matrix} X_N \\ X_I \end{matrix} \right\}$$

Where

T	loads transformation
X_N	non-interface payload displacements
X_I	interface displacements

Then derive $\left\{ X \right\}$ in terms of applied and inertial loading

$$\left\{ \begin{matrix} X_N \\ X_I \end{matrix} \right\} = \left[\begin{array}{c|c} K_{NN}^{-1} & 0 \\ \hline 0 & 0 \end{array} \right] \left\{ P_S \right\} + \left[\begin{matrix} \phi_c \\ I \end{matrix} \right] \left\{ X_I \right\}$$

Where

K_{NN}^{-1}	influence coefficients with interface grounded
P_S	applied or inertial loads
ϕ_c	constraint modes relating payload to interface motion

The member loads equation becomes

$$\left\{ML\right\} = \underbrace{\left[T\right]\left[\begin{array}{c|c} K_{NN}^{-1} & 0 \\ \hline 0 & 0 \end{array}\right]}\left\{P_s\right\} + \left[T\right]\begin{bmatrix} \phi_c \\ I \end{bmatrix}\left\{X_I\right\}$$

can be formed by application of unit 1# loads to payload with fixed interface and calculating payload inertial loads. Unit loads are applied one at a time to each payload dof to build total matrix.

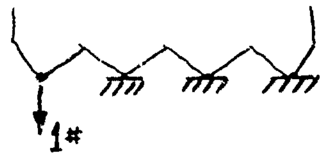
Then final LT1 can be formed as

$$\underbrace{\left[T\right]\left[\begin{array}{c|c} K_{NN}^{-1} & 0 \\ 0 & 0 \end{array}\right]}_{\text{from unit load sclution}}\left[-M_{NN}\right]\begin{bmatrix} \phi_N & \phi_c \\ & I \end{bmatrix}$$

The other side of the LLM equation represents member loads due to interface displacement,

$$\begin{bmatrix} LT2 \end{bmatrix} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} \phi_c \\ I \end{bmatrix}$$

Where $\phi_c = -K_{22}^{-1} K_{21}$ and is usually formed by making this matrix product. However, ϕ_c can also be formed by "unit load" solution applied to interface dof.



Unit loads are applied to one dof of the interface at a time with all other interface dofs fixed. Displacements of the payload and internal loads are computed. Normalizing these loads and displacements to a unit 1" interface displacement forms one column in

$$\underbrace{\begin{bmatrix} T \end{bmatrix} \begin{bmatrix} \phi_c \\ I \end{bmatrix}}_{\begin{bmatrix} T^* \end{bmatrix}} \quad \text{and} \quad \begin{bmatrix} \phi_c \\ I \end{bmatrix}$$

Continuing this procedure completes the required matrices without performing the inversion of K_{NN}

The reduced stiffness and mass matrices can be formed as

$$\left[K_I\right] = \left[K_{IN}\,\phi_c + K_{II}\right]$$

$$\left[M_I\right] = \begin{bmatrix}\phi_c \\ I\end{bmatrix}' \left[M_N\right] \begin{bmatrix}\phi_c \\ I\end{bmatrix}$$

or

$$= \left[\phi_c'\right]\left[M_{NN}\right]\left[\phi_c\right] + \left[M_{II}\right] \quad \text{(if } M_N \text{ is diagonal)}$$

SUMMARY

Unit load solutions can be used to form the "Revised" Methodology loads transformation portions:

$$\begin{bmatrix} T \end{bmatrix} \begin{bmatrix} K_{NN}^{-1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -M \end{bmatrix} \begin{bmatrix} \phi_N & \phi_c \\ 0 & I \end{bmatrix}$$

$$\begin{bmatrix} T \end{bmatrix} \begin{bmatrix} \phi_c \\ I \end{bmatrix}$$

Without inversion of the large matrix K_{NN} and with the additional advantage of obtaining ϕ_c, M_I, K_i . In other words, unit load solution replaces the collapsing procedure.